MECHANICS

Lecture No.7
Centroid

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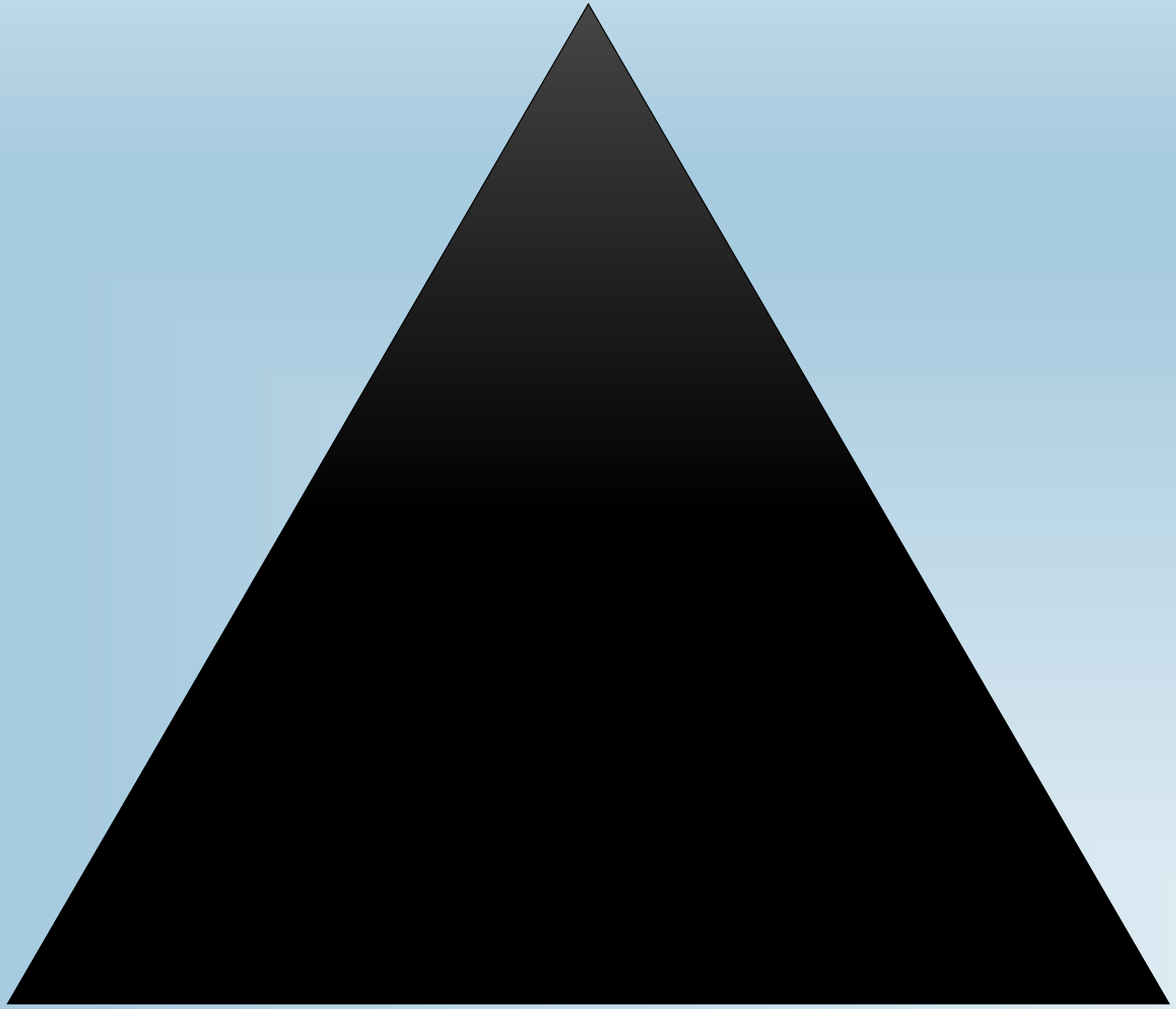
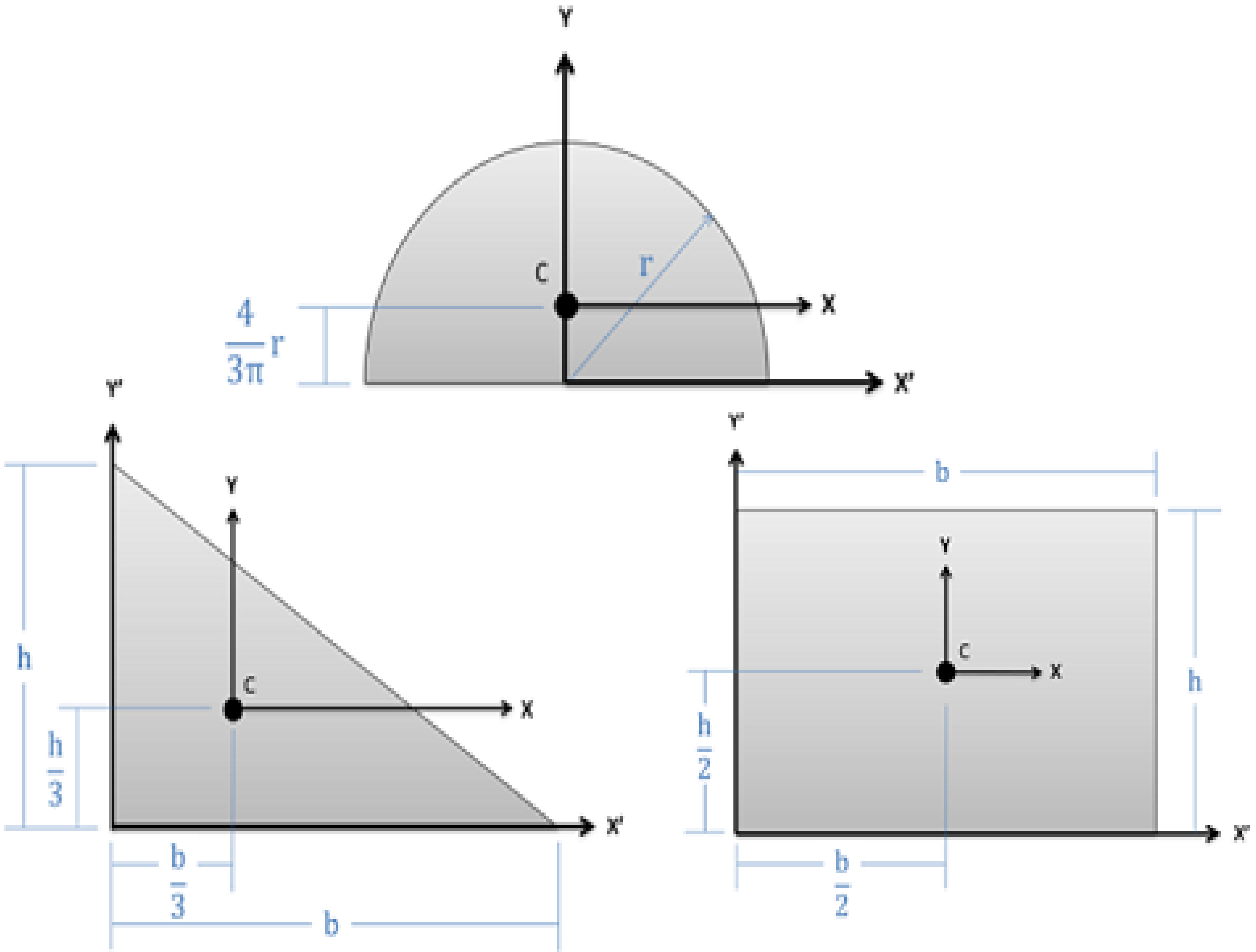
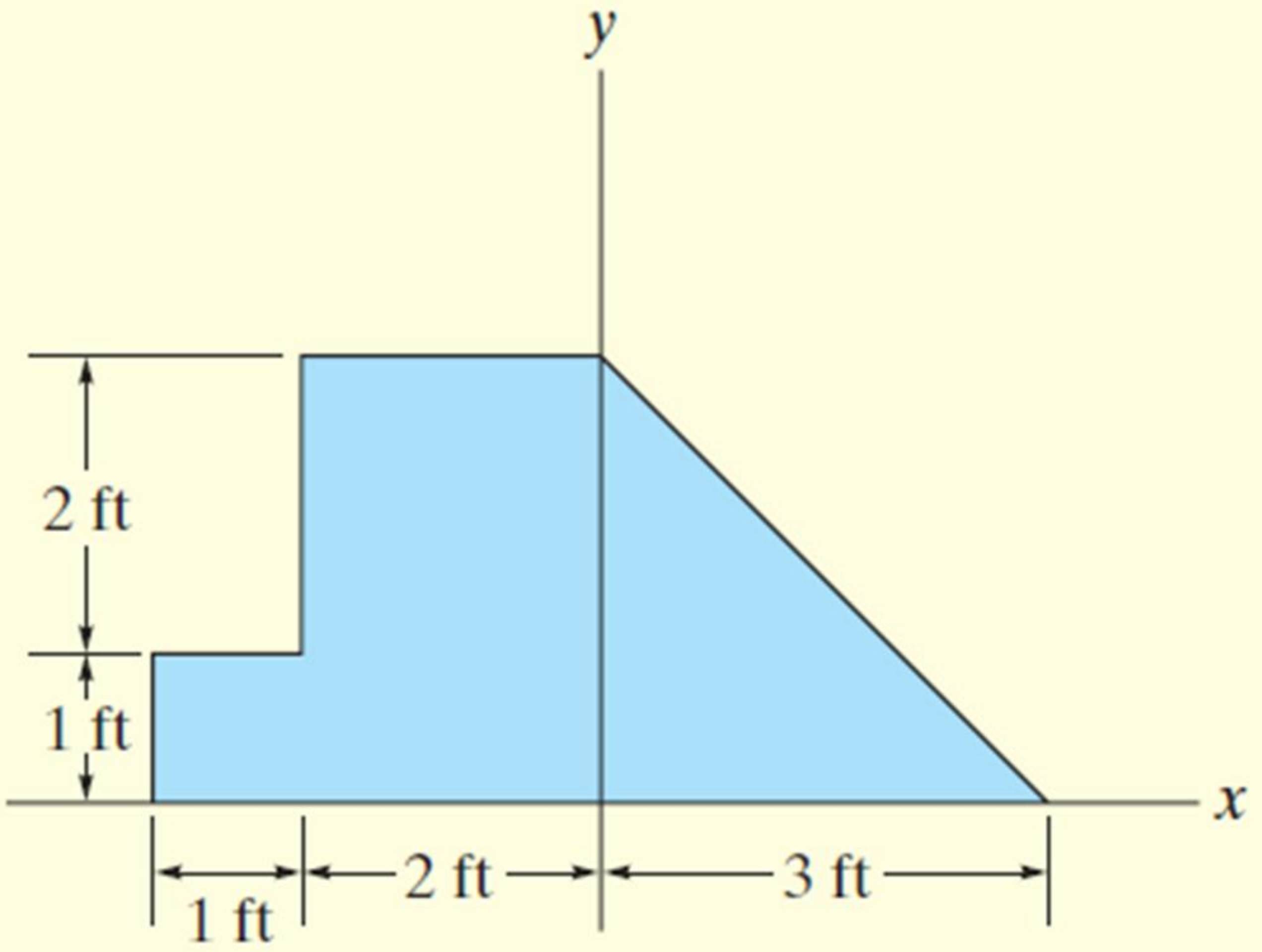
Centroid

The **centroid** of an area can be thought of as the geometric center of that area. The location of the centroid is often denoted with a 'C' with the coordinates being $\bar{x}$ and $\bar{y}$, denoting that they are the average x and y coordinate for the area. If an area was represented as a thin, uniform plate, then the centroid would be the same as the center of mass for this thin plate.

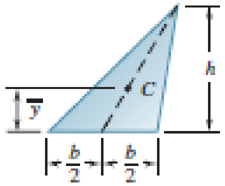
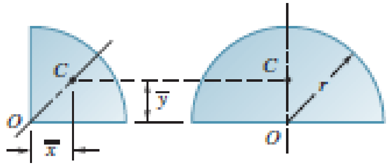
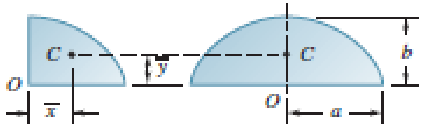
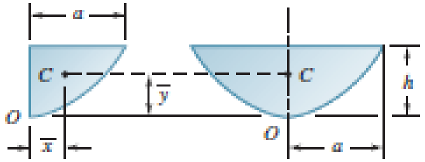
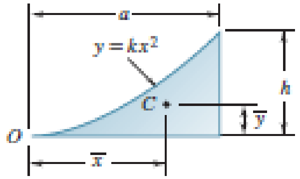
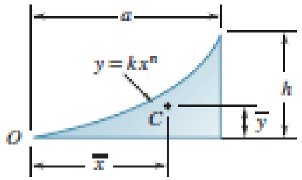
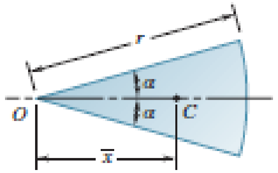


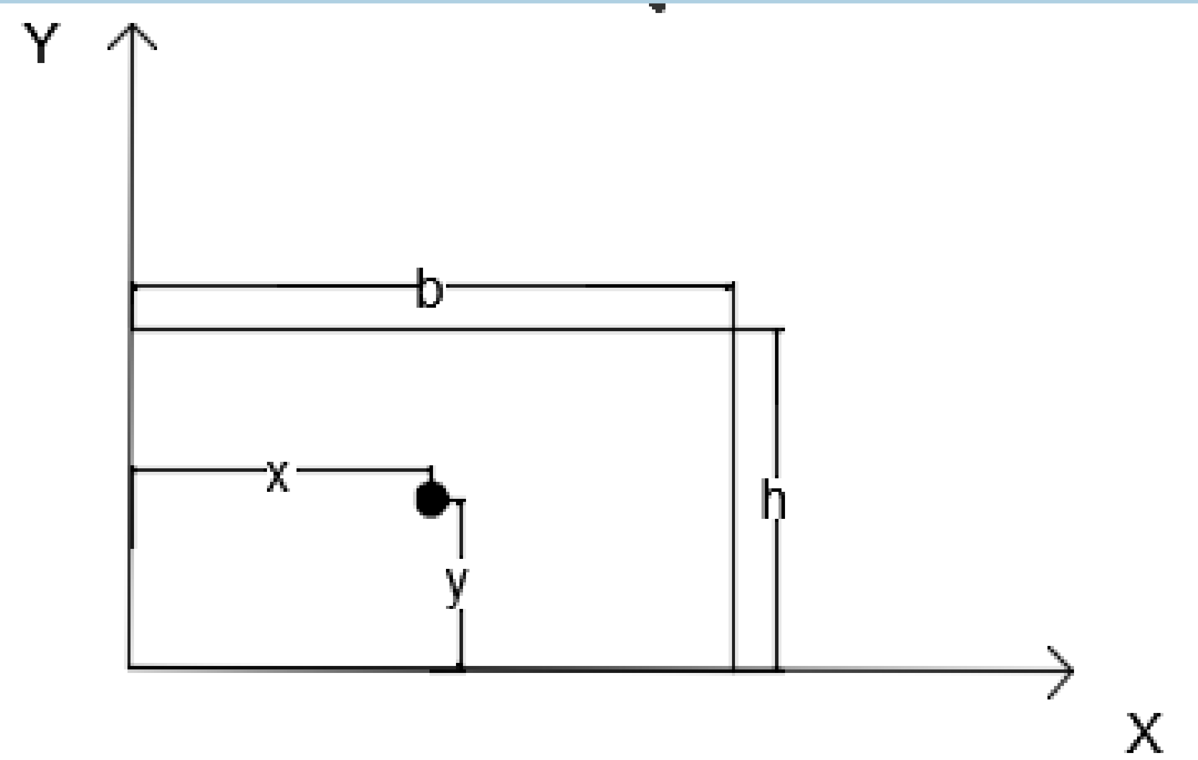

Y
C
r
X
4
3π r
X'
Y'
Y
h
C
X
h
3
X'
b
3
b
Y'
b
Y
C
X
h
h
2
X'
b
2



(a)

Centroids of areas are useful for a number of situations in the mechanics course sequence, including the analysis of distributed forces, the analysis of bending in beams, the analysis of torsion in shafts, and as an intermediate step in determining moments of inertia.

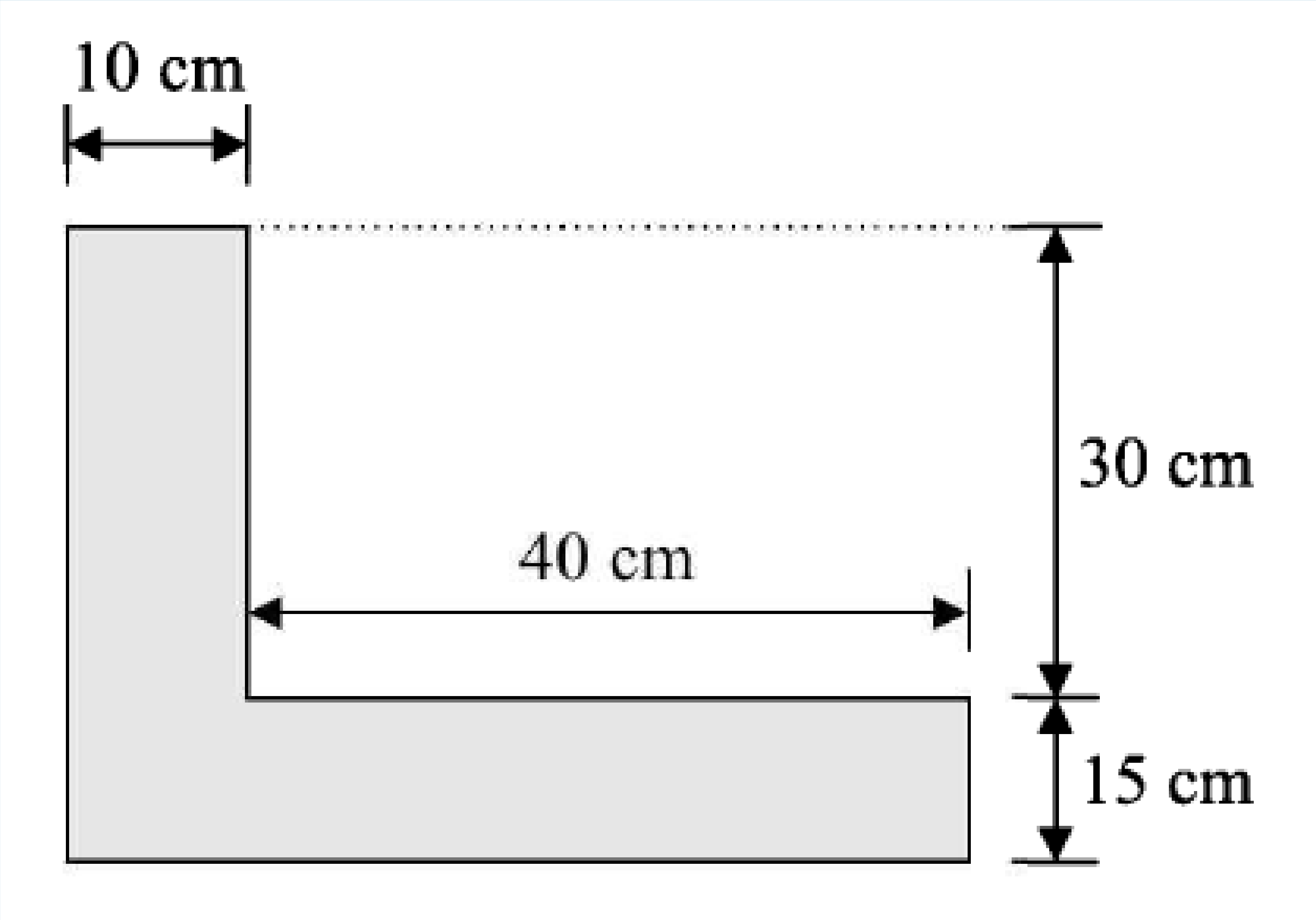
Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

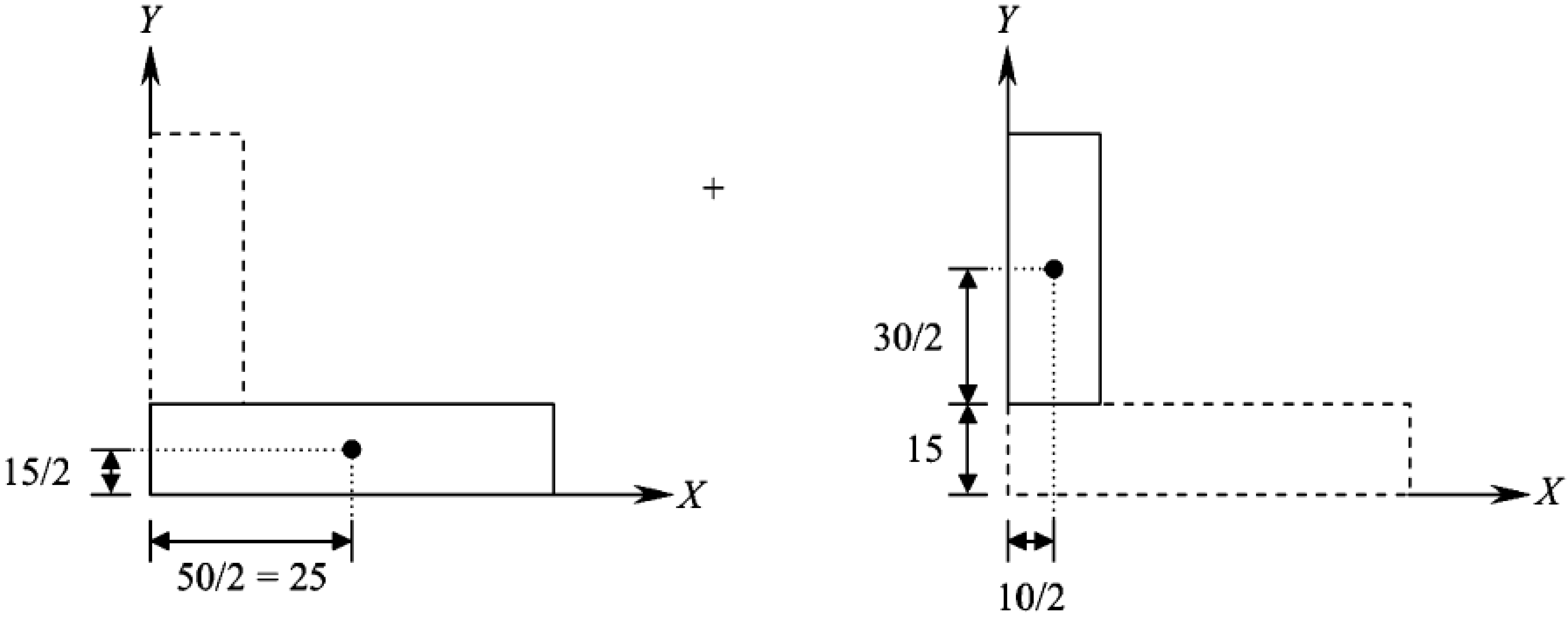


$$M_y = A\,.\,x \qquad\qquad x = \frac{M_y}{A}$$

$$M_x = A\,.\,y \qquad\qquad y = \frac{M_x}{A}$$

Locate the centroid of the plate area shown in Fig.




Y
X
15/2
50/2 = 25
+
Y
X
30/2
15
10/2

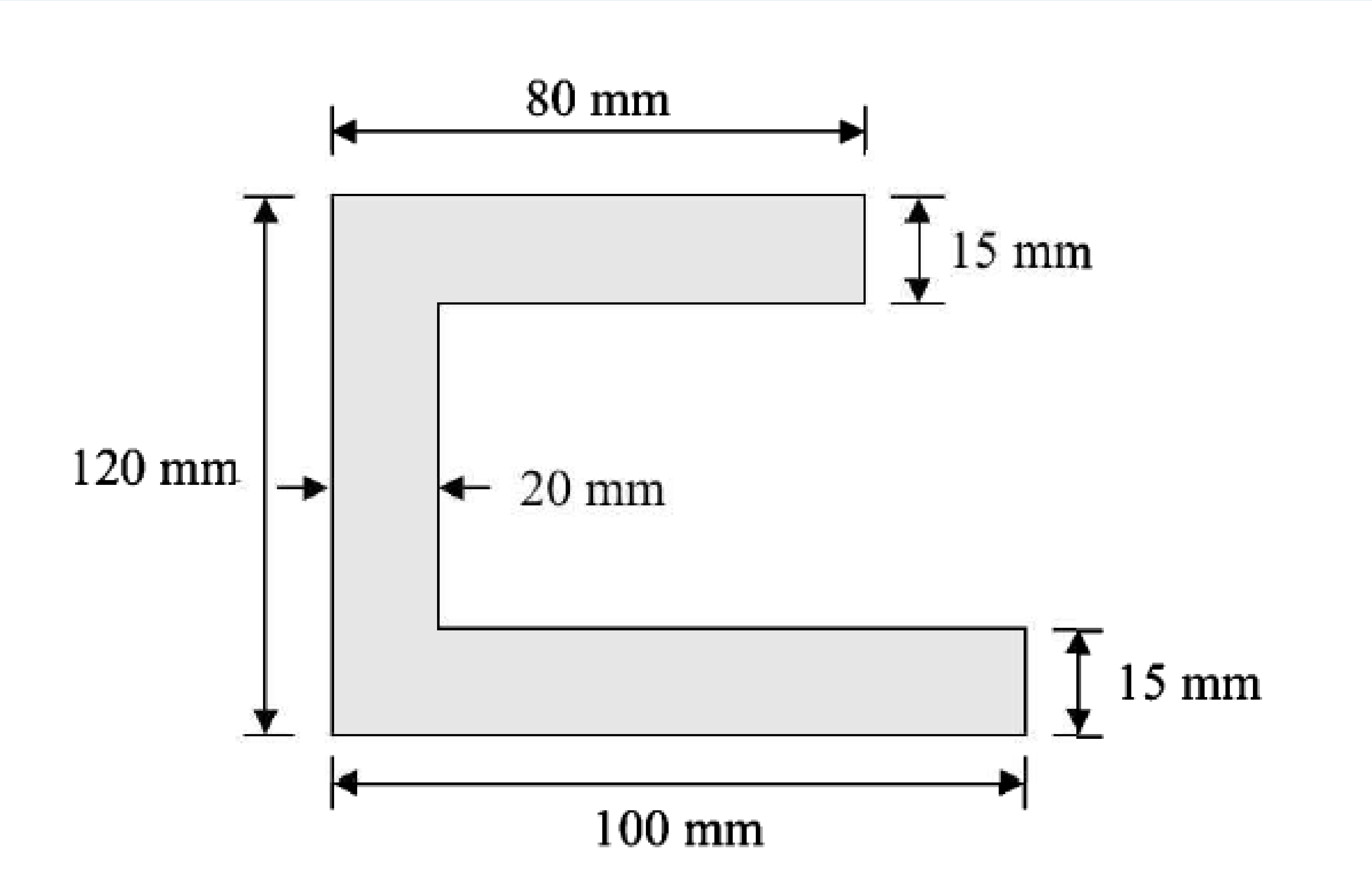
S.No	Element	A_i (cm^2)	$\bar{x}_i$ (cm)	$\bar{y}_i$ (cm)	$A_i\bar{x}_i$ (cm^3)	$A_i\bar{y}_i$ (cm^3)
1.	Rectangle-(1)	$50 \times 15 = 750$	$50/2 = 25$	$15/2 = 7.5$	18 750	5 625
2.	Rectangle-(2)	$10 \times 30 = 300$	$10/2 = 5$	$15 + (30/2) = 30$	1500	9000
$\Sigma =$		1050			20 250	14 625

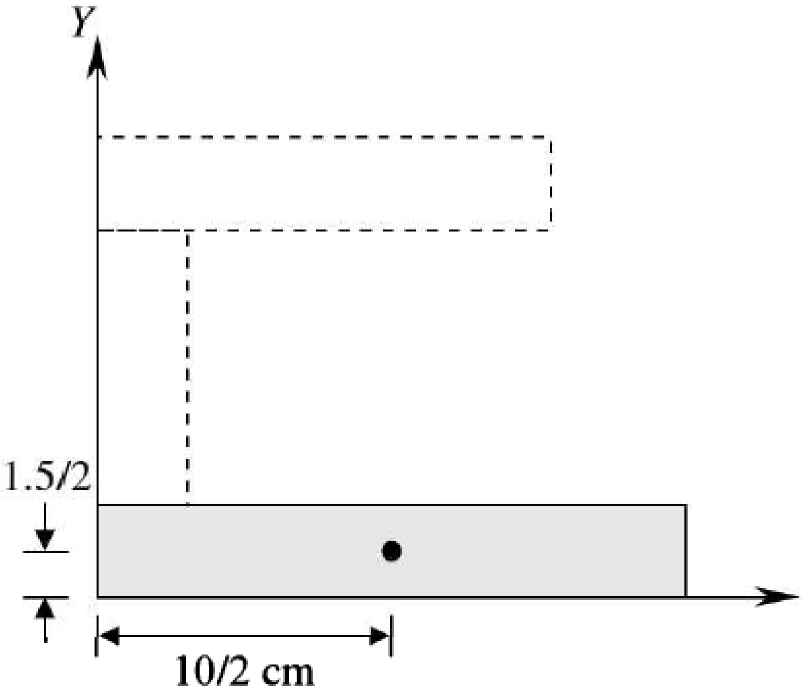
Therefore, the coordinates of the centroid are

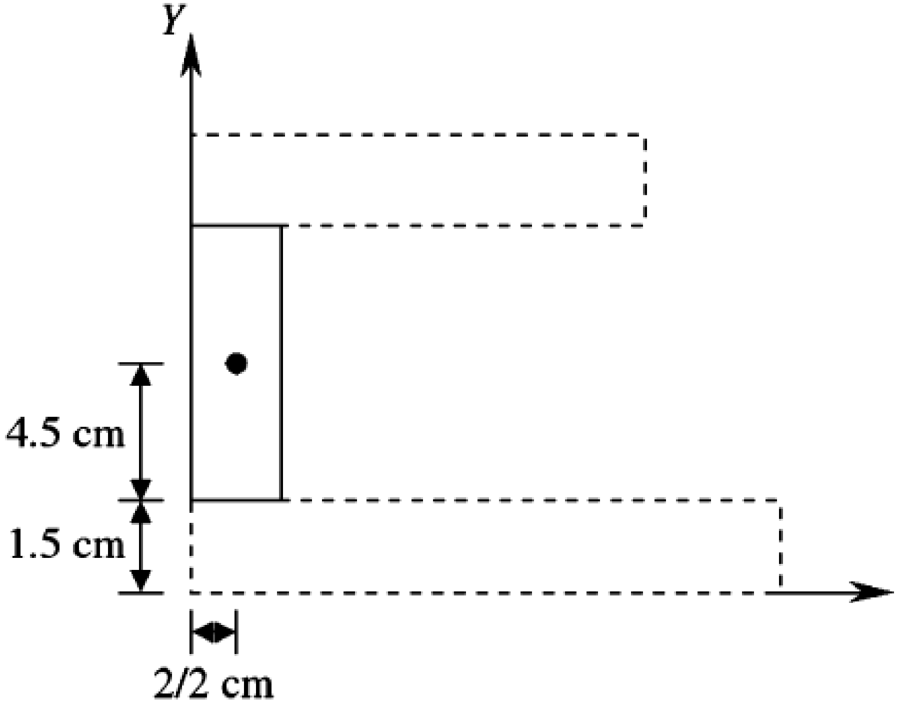
$$\bar{x} = \frac{\sum A_i\bar{x}_i}{\sum A_i} = \frac{20\,250}{1050} = 19.29 \text{ cm}$$

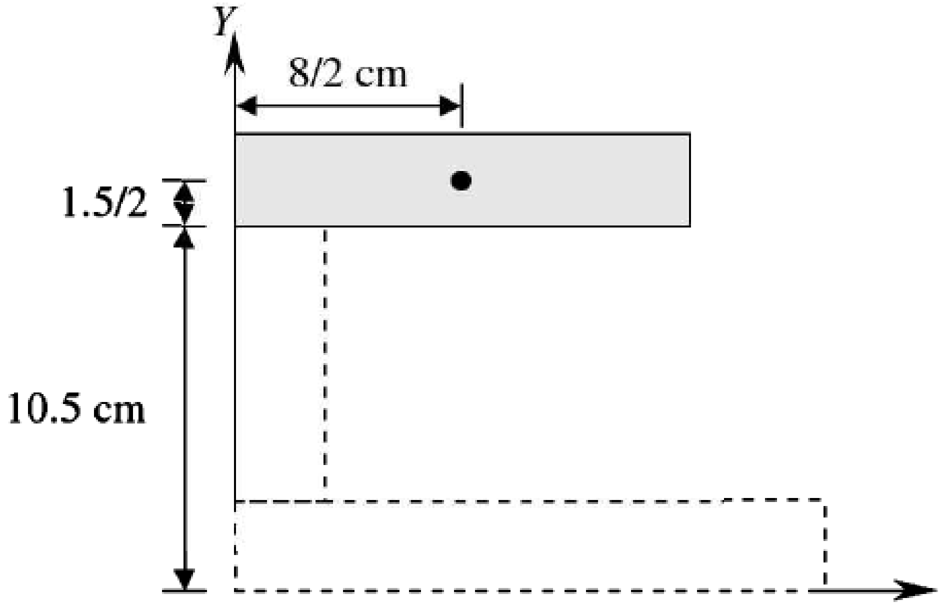
$$\bar{y} = \frac{\sum A_i\bar{y}_i}{\sum A_i} = \frac{14\,625}{1050} = 13.93 \text{ cm}$$

Locate the centroid of the plate area shown in Fig.




Y
1.5/2
10/2 cm


Y
4.5 cm
1.5 cm
2/2 cm


Y
8/2 cm
1.5/2
10.5 cm

$S.No$	$Element$	$A_i\ (cm^2)$	$\bar{x}_i\ (cm)$	$\bar{y}_i\ (cm)$	$A_i\bar{x}_i\ (cm^3)$	$A_i\bar{y}_i\ (cm^3)$
1.	Rectangle-(1)	$10 \times 1.5 = 15$	$10/2 = 5$	$1.5/2 = 0.75$	75	11.25
2.	Rectangle-(2)	$[12 - 2(1.5)] \times 2 = 18$	$2/2 = 1$	$12/2 = 6$	18	108
3.	Rectangle-(3)	$8 \times 1.5 = 12$	4	$12 - 1.5/2 = 11.25$	48	135
	$\Sigma =$	45			141	254.25

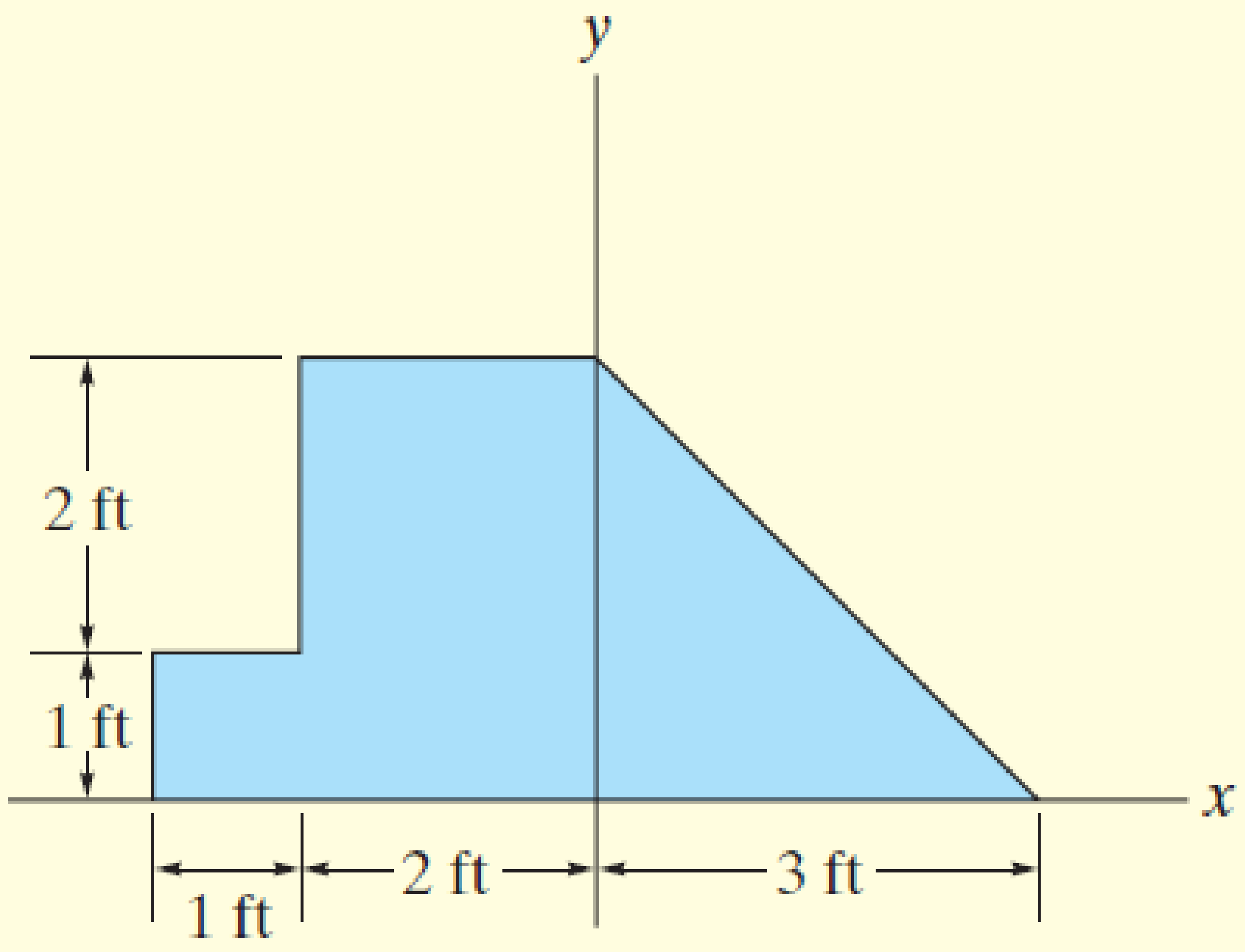
Therefore, the coordinates of the centroid are

$$\bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i} \qquad \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i}$$

$$= \frac{141}{45} \qquad = \frac{254.25}{45}$$

$$= 3.13 \text{ cm (or) } 31.3 \text{ mm} \qquad = 5.65 \text{ cm (or) } 56.5 \text{ mm}$$

Locate the centroid of the plate area shown in Fig.



(a)

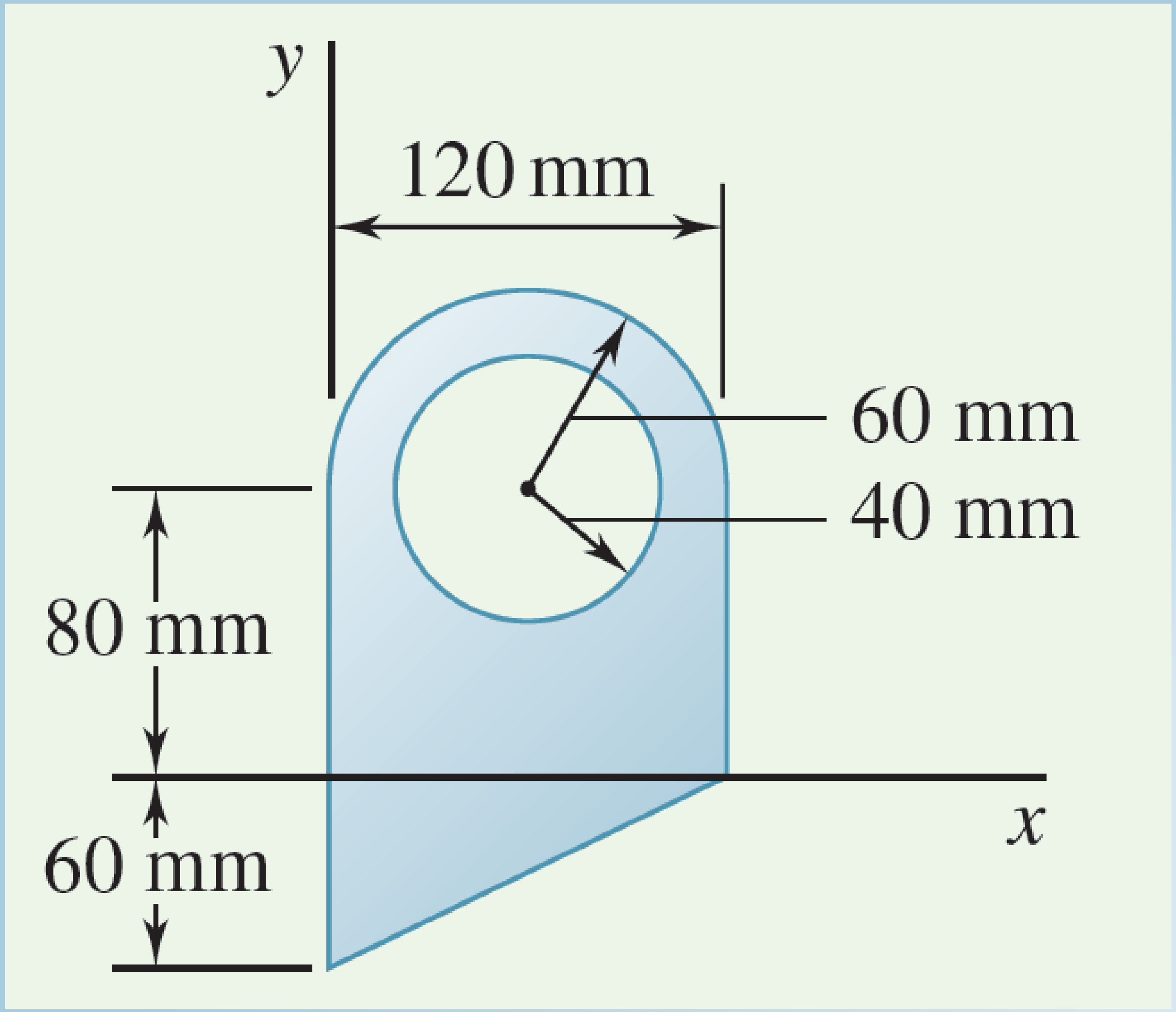
Segment	A (ft^2)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}A$ (ft^3)	$\tilde{y}A$ (ft^3)
1	$\frac{1}{2}(3)(3) = 4.5$	1	1	4.5	4.5
2	$(3)(3) = 9$	-1.5	1.5	-13.5	13.5
3	$-(2)(1) = -2$	-2.5	2	5	-4
	$\Sigma A = 11.5$			$\Sigma \tilde{x}A = -4$	$\Sigma \tilde{y}A = 14$

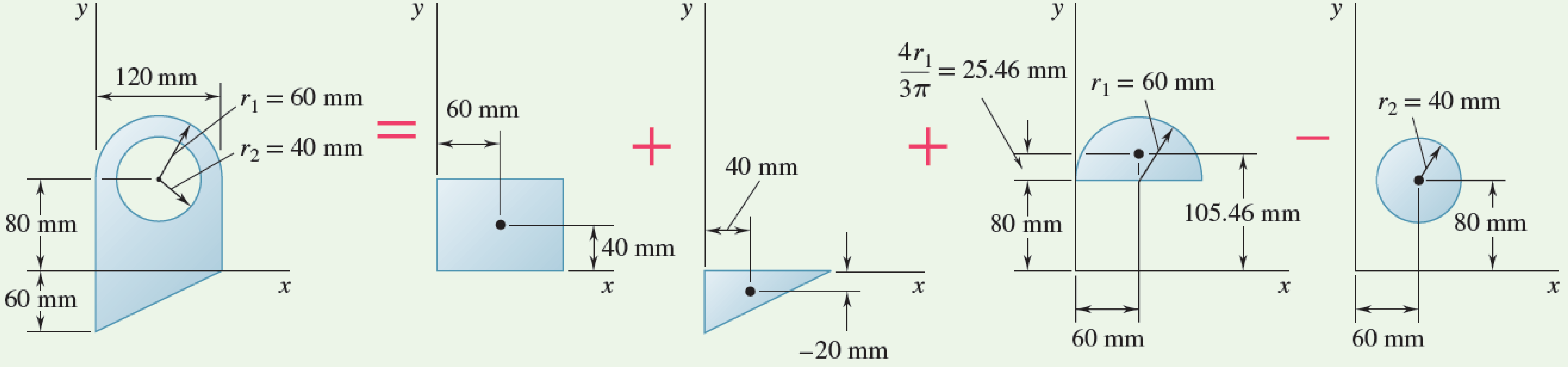
Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348 \text{ ft} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22 \text{ ft} \qquad \textit{Ans.}$$

For the plane area shown, determine (a) the first moments with respect to the x and y axes; (b) the location of the centroid.





Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	-20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

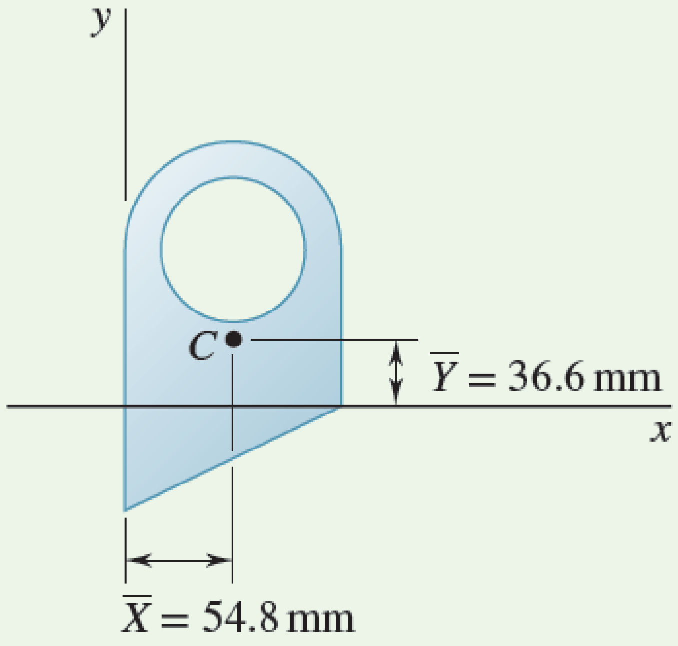


Fig. 2 Centroid of composite area.

ANALYSIS:

a. First Moments of the Area. Using Eqs. (5.8), you obtain

$$Q_x = \Sigma \overline{y}A = 506.2 \times 10^3 \text{ mm}^3 \qquad Q_x = 506 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

$$Q_y = \Sigma \overline{x}A = 757.7 \times 10^3 \text{ mm}^3 \qquad Q_y = 758 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

b. Location of Centroid. Substituting the values given in the table into the equations defining the centroid of a composite area yields (Fig. 2)

$$\overline{X}\Sigma A = \Sigma \overline{x}A: \qquad \overline{X}(13.828 \times 10^3 \text{ mm}^2) = 757.7 \times 10^3 \text{ mm}^3$$

$$\overline{X} = 54.8 \text{ mm} \blacktriangleleft$$

$$\overline{Y}\Sigma A = \Sigma \overline{y}A: \qquad \overline{Y}(13.828 \times 10^3 \text{ mm}^2) = 506.2 \times 10^3 \text{ mm}^3$$

$$\overline{Y} = 36.6 \text{ mm} \blacktriangleleft$$

Thank you for listening